19. (1,1) $L=\int_1^4 \sqrt{1+\frac{1}{4x}}\,dx$

d.e. $\frac{dy}{dx}=\sqrt{\frac{1}{4x}}$

$y=\int\sqrt{\frac{1}{4x}}\,dx$

$y=\sqrt{x}+c$

$1=1+c$

$y=\sqrt{x}$

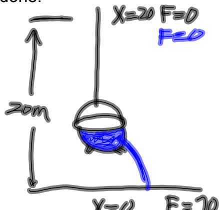
Jan 7-7:41 AM

7.5a Applications from Science and Statistics

Work done by a variable force: $W=\int_a^b F(x)\,dx$

Dec 17-7:20 PM

A leaky bucket weighs 22N empty. It is lifted from the ground at a constant rate at a point 20m above the ground by a rope weighing 0.4 N/m. The bucket starts with 70N of water but it leaks at a constant rate and just finishes draining as the bucket reaches the top. Find the amount of work done.




$W=Fd$ if F is constant

bucket $W=22N\cdot 20m = 440J$

water $F=-\frac{7}{2}(x-0)+70$

$F=-\frac{7}{2}x+70$

$m=\frac{70-0}{0-20}=-\frac{7}{2}$

$W=\int_0^{20}-\frac{7}{2}x+70\,dx$

$=700J$

$m=\frac{8-0}{0-20}=-.4$

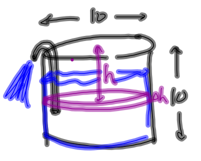
rope: $W=\int_0^{20}-.4(x-0)+8$

$=80J$

$440+700+80=1220J$

Dec 17-7:25 PM

How much work does it take to pump all the water over the rim of a cylindrical tank of height 10ft and diameter 10ft?




ΔW = work to lift one slice

ΔF = force to lift one slice

$\Delta W=\Delta F\cdot d=\Delta F\cdot h$

$\Delta F$ = weight = weight density · $\Delta V$

$=62.4\frac{lb}{ft^3}\cdot\pi\cdot 5^2\Delta h\ ft^3$

$\Delta V=\pi r^2\Delta h$

$\Delta V=\pi\cdot 5^2\Delta h$

$\Delta W=\underbrace{62.4\cdot\pi\cdot 5^2\cdot\Delta h}_{force}\cdot \underbrace{h}_{dist.}$

$\Delta W=62.4\pi\cdot 5^2 h\,\Delta h$

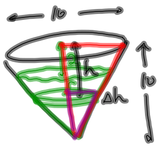
$W=\lim_{\Delta h\to 0}\sum 62.4\pi\cdot 5^2 h\Delta h$

$W=\int_0^{10}62.4\cdot\pi\cdot 5^2 h\,dh$

$=245044$ ft-lbs

Dec 17-7:29 PM

A conical tank of height and diameter 10ft is filled to within 2 ft of the top with olive oil weighing 57 lb/ft^3. How much work does it take to pump the oil to the rim of the tank?




$$\frac{5}{10} = \frac{r}{10-h}$$

$$r = \frac{1}{2}(10-h)$$

$$\Delta W = \Delta F \cdot h$$

$$\Delta F = 57\frac{lb}{ft^3} \cdot (\pi r^2 \cdot \Delta h)\, ft^3$$

$$\Delta F = 57\pi\left[\frac{1}{2}(10-h)\right]^2 \Delta h$$

$$\Delta W = 57\pi\left[\frac{1}{2}(10-h)\right]^2 \cdot h\, \Delta h$$

$$W = \int_2^{10} 57\pi\left\{\frac{1}{2}(10-h)\right\}^2 h\, dh$$

Dec 17-7:31 PM

Jan 7-8:25 AM